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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

For Supervisor's use only

Level 3 CAS Calculus, 2009

90834 Demonstrate an understanding of equations and expressions when solving problems

Credits: Seven
2.00 pm Thursday 26 November 2009

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Demonstrate an understanding of equations and expressions when solving problems.	Demonstrate a deeper understanding of equations and expressions when solving problems.	Demonstrate a comprehensive understanding of equations and expressions when solving problems.
Overall Level of Performance		

You are advised to spend 65 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

(a) Given complex numbers $v = 5 + ki$ and $w = 5 - ki$, Sam found vw as follows.

$$\begin{aligned} vw &= (5 + ki)(5 - ki) \\ &= 25 + 5ki - 5ki - k^2i^2 \\ &= 25 - k^2 \end{aligned}$$

Find and explain the error Sam has made.

Show the correct working.

(b) Solve exactly $5^{3n+2} = 6$.

Give your answer in terms of base 5 only.

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(c) Show that the solution to $\log_a(bx + c) = d$ is $x = \dfrac{a^d - c}{b}$.

(d) Find all the solutions to $z^4 = mi$, where m is a positive real number.

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(e) An extension ladder makes an angle of $\alpha°$ with the ground when leaning against a vertical wall.

It reaches a height of h metres up the wall.

The lower end of the ladder is d metres from the base of the wall.

When the ladder is extended, with the lower end of the ladder in the same position, it makes an angle of $2\alpha°$ with the ground and it reaches x metres further up the wall.

Show that an expression for x in terms of α and d only is $x = \dfrac{d\tan\alpha(1+\tan^2\alpha)}{1-\tan^2\alpha}$.

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(f) u and v are two complex numbers, $v \neq 0$, such that $|u+v|^2 = |u|^2 + |v|^2$.

Prove that $\frac{u}{v}$ is purely imaginary.

QUESTION TWO

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(a) Chris writes $\left(4+\sqrt{k}\right)\left(9-2\sqrt{k}\right)$ as $36-\sqrt{k}-2k$.

Find and explain the error in Chris's working.

Show the correct working.

(b) Find all solutions in radians for $4\tan 3(x-\frac{\pi}{8})=1$ where $0 \le x \le \frac{\pi}{2}$.

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(c) $z = -4 + 5i$ is one solution of the equation

$$z^3 + Az^2 + 17z - 123 = 0$$

Find the value of A.

(d) Prove that $\sin a \cos(a + b) - \cos a \sin(a + b) = -\sin b$.

(e) Solve the following equation for x in terms of k.

$$\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{k}{6}$$

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Extra paper for continuation of answers if required.
Clearly number the question.

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Question number

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